Systems at zero temperature. Third law of thermodynamics

A generic system has quantised energy levels

$\varepsilon_0, \varepsilon_1, \varepsilon_2, \dots$

Consider very low temperatures $T \ll \Delta\varepsilon = \varepsilon_1 - \varepsilon_0$

$E = \varepsilon_0$

Then $C_V = \left(\frac{\partial E}{\partial T}\right)_V = \left(\frac{\partial \varepsilon_0}{\partial T}\right)_V = 0$

Previously, we derived also that

$$C_p = C_V - T\frac{\left(\frac{\partial V}{\partial T}\right)_p^2}{\left(\frac{\partial V}{\partial p}\right)_T}$$

Therefore, at $T=0$ we also have $C_p = 0$

So, at $T=0$ $\quad C_p = C_V = 0$

<u>Entropy</u> $S = \frac{E}{T} + \ln Z = \frac{\varepsilon_0}{T} + \ln e^{-\frac{\varepsilon_0}{T}} = 0$

$S = 0$ – <u>Nernst theorem</u>

= <u>3rd law of thermodynamics</u>

(Using the the thermodynamic definition we could have said that it's an arbitrary constant)

$S \to \text{const}$

that

have

$S \to \text{const}$

Then the meaning of the theorem is that this const is independent of other macroscopic parameters

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From this theorem it follows also that

$$\left(\frac{\partial S}{\partial P}\right)_T = 0$$

$$\parallel \leftarrow \text{A Maxwell's relation}$$

$$-\left(\frac{\partial V}{\partial T}\right)_P$$

$\Longrightarrow$ The thermal expansion coefficient also vanishes, $\alpha = 0$, at $T = 0$

Note: there may be a violation of the 3rd law if there is a dedeneracy of the ground state.

The entropy of N qubits

$S = N \ln 2$